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GENERATION OF PRESSURE OSCILLATIONS IN AIR FLOWS IN VERTICAL CHANNELS WITH INTERNAL HEAT RELEASE

Abstract: *The characteristics of the air flow in a vertical channel, arising due to local internal heat release, are investigated by the method of numerical simulation. Heat is supplied to the flow from internal sources located in a limited volume closer to the inlet section of the channel. The problem of flow and heat transfer is described by a system of unsteady Navier-Stokes and energy equations for a compressible medium. The coefficients of viscosity and thermal conductivity are considered to be temperature dependent. From the numerical solution of this system, the velocity, pressure, and temperature fields in the channel are determined. Based on the results of the calculations, the regularities of the change in time of velocity and pressure in the channel are determined. From the analysis of the results it follows that from the moment the heat supply begins, a vertical air flow develops in the channel, which is accompanied by oscillations in velocity and pressure. Self-oscillations arising in a gas flow are a manifestation of instability of flow. It is shown that stable oscillations take place in the presence of additional local hydraulic resistance in the channel. The dependence of the amplitude and frequency of pressure oscillations and the air flow velocity on the power of the sources of internal heat release and the height of the channel has been investigated. It was determined that with an increase in the power of the source of internal heat supply and the height of the channel, the amplitudes of the velocity and pressure fluctuations increase.*

Keywords: *natural convection, numerical modeling, flow instability, pressure self-oscillations, oscillation amplitude.*

Introduction

Periodic self-oscillatory processes accompanying the supply of heat into the flow of a compressible medium is an important factor that must be taken into account when designing heat and power equipment. Thermoacoustic self-oscillations can occur when heat is supplied by convection to the flow from an external source or when fuel mixtures are burned in different thermal devices. Self-oscillations of pressure can create additional mechanical loads on the structure of the combustion device, which can lead to its damage. This regime can also change the heat transfer conditions. Therefore, an important problem is to determine the dependence of the characteristics of self-oscillations on the level of thermal load on the system, as well as on the geometric characteristics and operating parameters of the furnace devices.

Thermoacoustic self-oscillations arising in a gas flow is one of the manifestations of flow instability. An unstable flow regime occurs in the elements of power equipment, if there is an internal release of heat in a limited volume of it, or there is an external supply of heat. One of the well-known processes accompanied by the generation of thermoacoustic self-oscillations is vibrational combustion. This phenomenon is observed during the combustion of fuel mixtures in air heaters for blast furnaces, in the combustion chambers of jet engines and in a number of other devices. At certain frequencies and amplitudes, such oscillations can become dangerous to equipment and can lead to its destruction. Self-oscillations of pressure do not allow increasing the power and economic performance of power equipment. The regularities of this phenomenon have not yet been sufficiently studied. Therefore, there are problems with the definition of ways to prevent its destructive action. It follows from this that the problem of the occurrence of pressure self-oscillations in the flow of coolants is urgent today.

The Rijke tube is often used as an object for studying thermoacoustic oscillations. Rijke tube is a channel with a located heat source. Acoustic oscillations are generated in such channel under certain conditions. The characteristics of these oscillations in the Rijke tube were investigated experimentally and theoretically on the basis of a number of mathematical models. A detailed review and analysis of the results of analytical and experimental studies related to the Rijke tube, carried out before 1993, is presented in [1]. The authors of [2] associate the relevance of research on self-oscillations in devices like the Rijke tube with problems arising in the combustion chambers of jet and rocket engines. Another problem, in connection with which the study of the Rijke phenomenon is relevant, is associated with processes in pulsed combustion chambers and coal bed combustors. In [3] a computational fluid dynamics technique is used to investigate the unsteady flow field inside a Rijke tube. This investigation is carried out to explain the coupling that exists in such environment between heat addition, pressure and velocity oscillations. It is shown that the location of the heat source is a key factor in producing oscillations inside the Rijke tube. It was found that pressure oscillations appear to grow exponentially with increasing of heat input.

Theoretical studies of thermoacoustic oscillations were carried out mainly at simplified mathematical formulations of problems of hydrodynamics and heat transfer, which make it possible to obtain their analytical solutions. In [4], on the basis of a simplified mathematical model, including one-dimensional equations for the transfer of mass, momentum and energy, an analysis of the characteristics of acoustic oscillations in a Rijke tube with internal heat supply from electrically heated grid is carried out. The mathematical model proposed in [4] for the Rijke tube is recommended to be used for preliminary design and analysis of real thermal devices, where thermoacoustic instability may occur. In [5], using a similar one-dimensional mathematical model, the thermoacoustic instability in the Rijke tube was investigated. By using the Galerkin approximation, a thermoacoustic model was built, on the basis of which the study of velocity and pressure perturbations was carried out. With the obtained research results, a control scheme for a combustion device similar to the Rijke tube has been developed.

In [6], the thermoacoustic instability in a Rijke tube with a distributed heat source is investigated by solving a system of one-dimensional equations of momentum and energy transfer. Due to the widespread of thermoacoustic instability in the combustion chambers of gas turbines, this work is aimed to study this phenomenon and to solve this problem. The heat release model consists of a number of distributed heat sources with individual heat release rates. This paper examines the influence of multiple heat sources, time delay of heat release on nonlinear system characteristics associated with a distributed heat source. It was found that a distributed heat source plays an important role in determining the stability of a thermoacoustic system.

The results of theoretical and experimental studies of thermoacoustic instability during combustion are presented in [7]. The work uses an approach to modeling based on a separate description of the hydrodynamic and acoustic fields. It is shown that this combined method is extremely effective, and the calculation results are confirmed by corresponding experiments.

In [8], the results of studies of pressure self-oscillations arising in combustion installations are presented. The theoretical research is based on the energy method. On its basis, a theoretical model has been developed that makes it possible to consider the self-excitation of longitudinal acoustic oscillations of a gas in typical combustion devices. In this case, it is believed that self-oscillations arise due to the presence of a phenomenological delay in the combustion process.

In [9] it is noted that pressure self-oscillations arise not only during combustion, but also during convective supply of heat into the flow. That is, in addition to the phenomenological delay in the combustion process, there are also other reasons for the excitation of self-oscillations, among which the so-called "negative" viscous friction resistance. It manifests itself in the form of a decrease, under certain conditions, of the frictional resistance in the channel with an increase in the flow velocity. Due to the fact that the viscosity of air increases with increasing temperature, an increase in the flow rate, which is accompanied by a decrease in temperature, can lead to a decrease in frictional resistance. In [10], the mechanisms of thermohydraulic instability of a gas flow with a local supply of heat were determined. The corresponding mathematical model of gas movement has been built. For the model under consideration, the thermal energy dissipation tensor is determined, which characterizes the presence of negative resistance. A simplified method for calculating the parameters of self-oscillations arising from unstable vibrational combustion in vertical combustion chambers of blast furnace stoves is described in [11].

In [12], the regularities of the occurrence of thermoacoustic instability inside a horizontal Rijke tube are investigated by an experimental method. The Rijke tube is equipped with a coaxial burner with gas premixing as a heat source, which can be placed in any position inside the tube. Methods for suppressing this instability are considered.

A generalization of the results of analytical studies of pressure self-oscillations obtained by solving the system of equations of motion of a continuous medium are presented in [13].

Since the considered research results were obtained using simplified mathematical models that make it possible to obtain analytical solutions, these results mainly describe the conditions for the occurrence and the qualitative nature of self-oscillations. To determine the quantitative characteristics of self-oscillations, it is necessary to apply more precise formulations of the problems of hydrodynamics and heat transfer and numerical methods for their solution. This research is carried out in order to determine the regularities of the occurrence of pressure self-oscillations in the flow of a compressible medium during local heat supply.

Materials and methods

To determine the characteristics of self-oscillations of pressure in the flow of a compressible medium, numerical modeling of the dynamics of air flow and energy transfer in a vertical channel with local internal supply of heat to the flow is performed. Air flow occurs due to natural convection arising from the internal local supply of heat to the air medium. Heat is supplied over a certain section of limited length Δz due to the action of internal sources. This section is located closer to the inlet cross section of the channel.

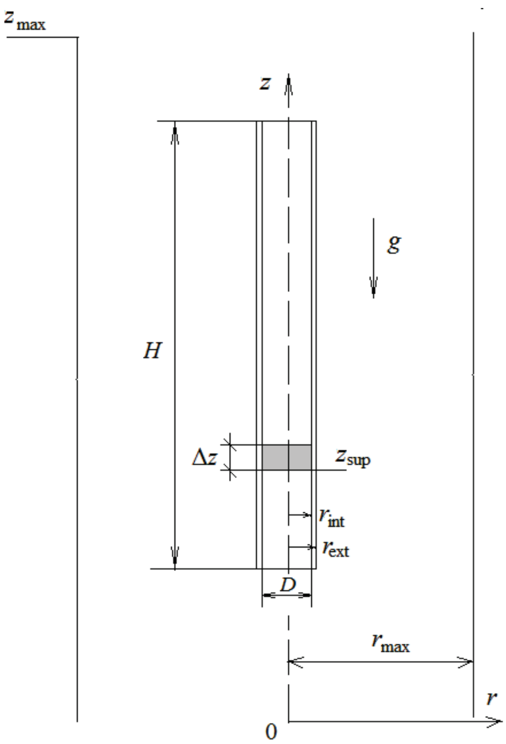


FIGURE 1. *Scheme of computational domain*

Numerical studies of hydrodynamics and heat transfer in a vertical channel, as well as an analysis of the possibility of thermohydraulic instability in the flow, are carried out in the computational domain (fig. 1), which includes a cylindrical channel, which is located in a cylindrical cavity with open upper and lower sections. Cavity radius – $r_{\max}$, cavity height – $z_{\max}$.

The movement of the compressible air medium in a cylindrical vertical channel is considered symmetrical about the axis of the cylinder. This motion is described by the system of Navier-Stokes and energy equations, which in polar coordinates has the form:

$$\frac{\partial \rho}{\partial \tau}+\frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r}+\frac{\partial(\rho v_z)}{\partial z}=0 \tag{1}$$

$$\begin{gathered}\frac{\partial(\rho v_z)}{\partial \tau}+\frac{1}{r}\frac{\partial(\rho r v_r v_z)}{\partial r}+\frac{\partial(\rho v_z^2)}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)\right)+\\ +\frac{\partial}{\partial z}\left(2\mu\frac{\partial v_z}{\partial z}-\frac{2\mu}{3}\left(\frac{1}{r}\frac{\partial(r v_r)}{\partial r}+\frac{\partial v_z}{\partial z}\right)\right)-\rho g\end{gathered} \tag{2}$$

$$\begin{gathered}\frac{\partial(\rho v_r)}{\partial \tau}+\frac{1}{r}\frac{\partial(\rho r v_r^2)}{\partial r}+\frac{\partial(\rho v_r v_z)}{\partial z}=-\frac{\partial p}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\left(2\mu\frac{\partial v_r}{\partial r}-\frac{2\mu}{3}\left(\frac{1}{r}\frac{\partial(r v_r)}{\partial r}+\frac{\partial v_z}{\partial z}\right)\right)\right)-\\ -\frac{1}{r}\left(2\mu\frac{v_r}{r}-\frac{2\mu}{3}\left(\frac{1}{r}\frac{\partial(r v_r)}{\partial r}+\frac{\partial v_z}{\partial z}\right)\right)+\frac{\partial}{\partial z}\left(\mu\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)\right)\end{gathered} \tag{3}$$

$$\frac{\partial(C_p\rho T)}{\partial \tau}+\frac{1}{r}\frac{\partial(r C_p\rho v_r T)}{\partial r}+\frac{\partial(C_p\rho v_z T)}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)+q_v \tag{4}$$

$$p=\rho R_a T \tag{5}$$

where:

τ – time;
r – radial coordinate;
z – vertical coordinate;
v_r – radial velocity;
v_z – vertical velocity;
p – pressure;
g – acceleration of gravity;
T – temperature,
C_p – heat capacity of the gas at constant pressure;
ρ – air density;
μ – dynamic coefficient of air viscosity;
λ – coefficient of thermal conductivity of air;
q_v – density of internal sources of heat;
R_a – gas constant for air.

Dynamic viscosity coefficient μ (T) and thermal conductivity of air λ (T) depend on temperature.

Boundary conditions are formulated for this system. It is considered that at $z = 0$ the temperature and pressure of the gas are known. It is also assumed that the radial velocity component in the lower section of the computational domain is zero. At the exit from the computational domain $z = z_{\max}$, the pressure is also known. The radial velocity component at $z = z_{\max}$ is taken to be zero. It is also assumed that at $z = z_{\max}$ the derivative of the temperature along the z coordinate is equal to zero. So, the boundary conditions for the system of equations (1)-(5) are formulated as:

$$z = 0:\ p = p_0;\ v_r = 0;\ T = T_\infty \tag{6}$$

$$z = z_{\max}:\ p = p_{\text{ex}};\ v_r = 0;\ \frac{\partial T}{\partial z} = 0 \tag{7}$$

The air pressure at the outlet from the computational domain p_{ex} is determined from the expression:

$$p_{ex} = p_0 \cdot \exp\left(-\frac{g}{R_a T_\infty} z_{\max} \right) \tag{8}$$

which follows from the condition of equality between the pressure drop between the lower and upper cross sections of the computational domain and the weight of the air column located between these sections.

On the axis of symmetry $r = 0$, as well as on the boundary of the computational domain $r = r_{\max}$, the following boundary conditions are accepted:

$$r = r_0;\ r = r_{\max}:\ v_r = 0;\ \frac{\partial v_z}{\partial r} = 0;\ \frac{\partial T}{\partial r} = 0 \tag{9}$$

On the channel surfaces, the following conditions are accepted:

$$r = r_{\text{int}};\ r = r_{\text{ext}}:\ v_r = 0;\ v_z = 0;\ \frac{\partial T}{\partial r} = 0 \tag{10}$$

The system of equations (1)-(5) with boundary conditions (6)-(10) is solved by the finite difference method. For this, a computational grid is constructed and the approximation of partial differential equations by finite differences is performed according to the scheme given in [14]. The system of finite difference equations is solved by the matrix method [15]. Based on the results of its solution, the fields of velocity, pressure and temperature are determined, which change in time.

Results

In figure 2 shows the velocity and temperature fields in a vertical channel with a height H =1.1 m and a diameter D = 0.1 m, obtained for the case of heat supply with a power Q = 3000 W from an internal volumetric source. The beginning of the heat supply section is at a distance of 0.16 m from the inlet cross section of the channel. The length of this section is 0.13 m. The results are given for three moments of time. It is assumed that at z = 0 the air temperature is T_∞ = 293 K, and the pressure is p_0 = 101000 Pa.

In figure 3 shows the changes in time of the air flow velocity in the inlet (fig. 3a) and in the outlet (fig. 3b) sections of the channel at a time interval of 1.45 s < τ < 1.50 s. As can be seen from this figure, the time variation of the velocity in these cross sections has the character of oscillations with variable amplitude. The average velocity in the inlet section reaches 1.04 m/s, and in the outlet section – 2.05 m/s. That is, the average velocity at the exit from the channel is twice the flow velocity at the entrance to the channel. The frequency of velocity oscillations is $\omega_v \sim 240$ Hz.

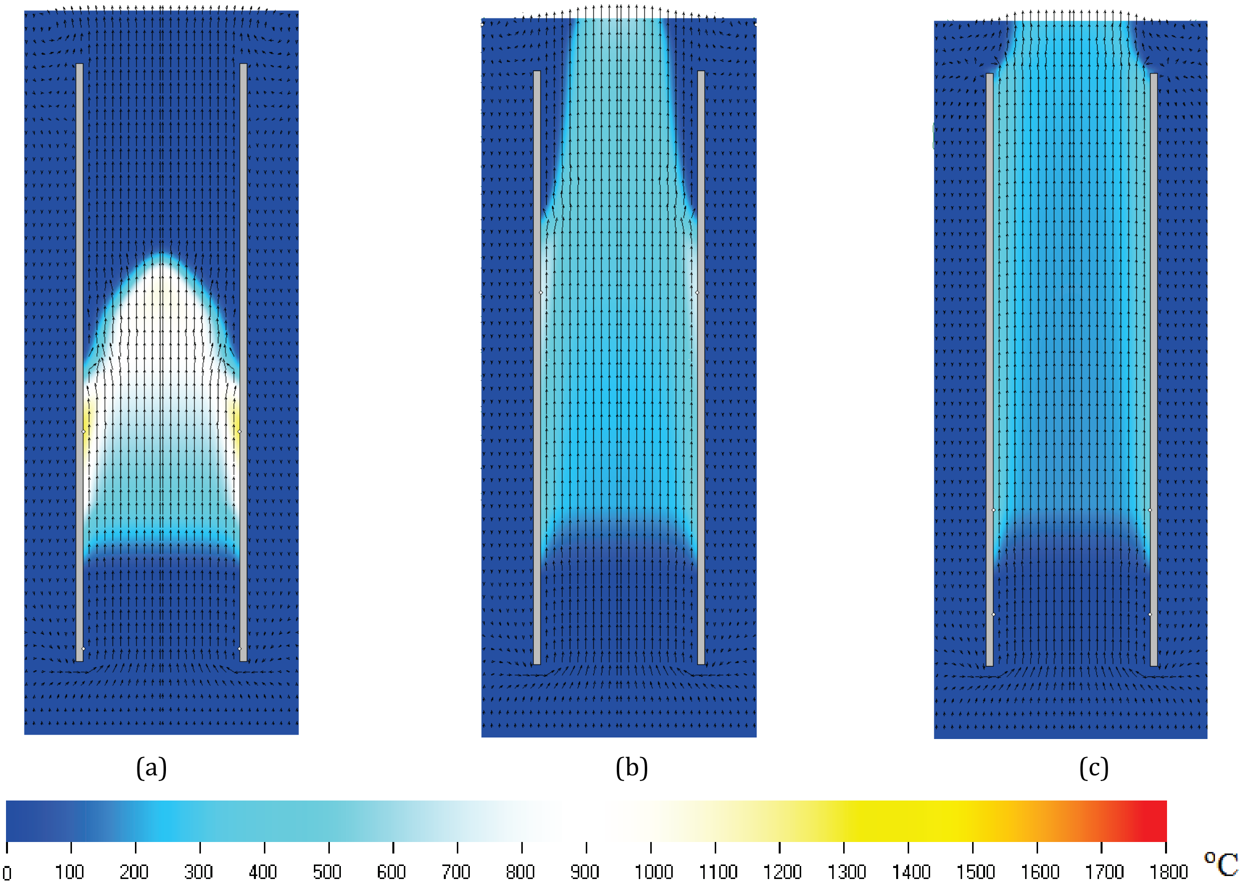


Figure 2. *Velocity and temperature fields in the vertical channel under the conditions of heat supply with power Q = 3000 W: a) τ = 1.0 s; b) τ = 1.5 s; c) τ = 2.0 s*

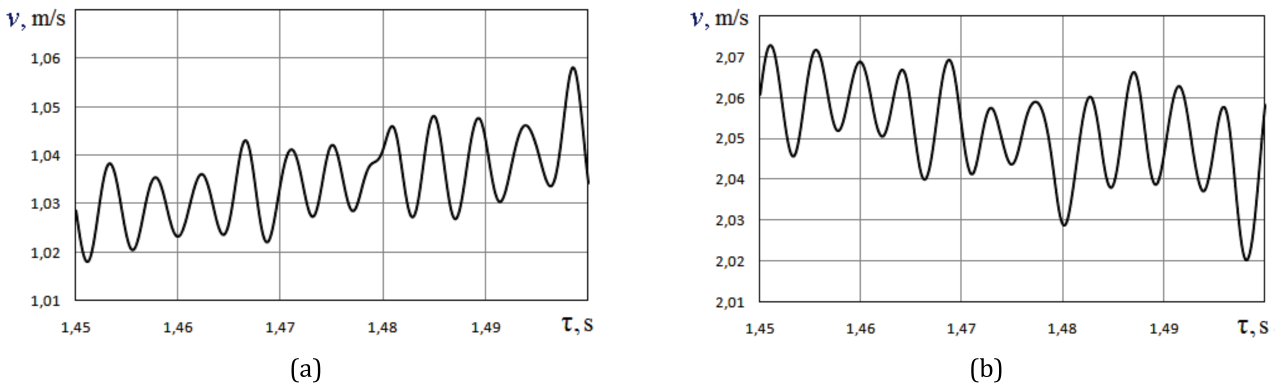


Figure 3. *Change of flow velocity in the inlet (a) and outlet (b) cross sections of the channel in the time interval 1.45 s < τ < 1.50 s under the conditions of heat supply with power Q = 3000 W*

A characteristic feature of the oscillatory motion of the gaseous medium in a vertical channel with a local heat supply is that the oscillations of the velocity at the exit from the channel are in antiphase with the oscillations of the velocity at the entrance to the channel. That is, at the maximum flow velocity at the outlet from the channel, the flow velocity at the inlet to the channel will be minimal, and at the minimum flow velocity at the outlet from the channel, the inlet velocity will be maximum. This can be seen from a comparison of the graphs shown in figure 3a and figure 3b. This indicates that the gas flow, moving upward under the action of the thermogravitational force, simultaneously performs oscillatory motion, expanding and narrowing in the direction of the inlet and outlet cross sections of the channel.

The time variation of the excess pressure in the channel cross section, which is located at a distance of 0.1 m from the outlet cross section, in the time interval 1.45 s < τ < 1.50 s are shown in figure 4. The excess pressure is measured from its value at z = 0, where it is equal to p_0. As can be seen from this figure, the time variation of the pressure in this section has the character of oscillations occurring with variable amplitude (from 1.5 Pa to 2 Pa) and with a frequency of $\omega_p \sim$ 240 Hz (with the same frequency as the velocity oscillations).

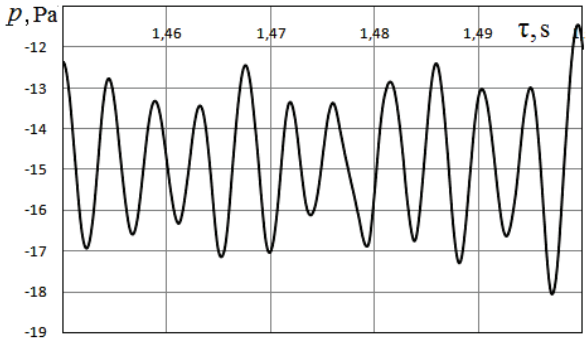


FIGURE 4. *Change of excess pressure in the cross section, located at a distance of 0.1 m from the outlet cross section of the channel, in the time interval of 1.45 s < τ < 1.50 s under the conditions of heat supply with power Q = 3000 W*

As follows from the results of calculations, the amplitudes of the velocity and pressure oscillations in the channel under the considered conditions decrease with time. After a while, the oscillations practically damp out due to dissipative effects.

In devices in which the combustion process of the fuel mixture (combustion chambers) takes place, there are usually separate elements that create additional local hydraulic resistance to the gas flow. Such elements can be, for example, flame stabilizers. Schematically, they can be represented as a system of concentric rings located in the channel and having a common axis of symmetry with the channel. The results of calculating the velocity and temperature fields under the same conditions that were considered earlier for a channel with a system of concentric rings near the outlet section of the channel are shown in figure 5. As can be seen from the comparison of figure 2 and figure 5, in the presence of these cylindrical rings, the velocity and temperature fields in the channel develop in time somewhat differently than in the absence of additional hydraulic resistance.

In figure 6 shows the time variation of the excess pressure in the channel section, located at a distance of 0.1 m from its outlet section. The results are compared for channels without local hydraulic resistance (1) and with local hydraulic resistance (2). The local heat release power is Q = 3000 W. From a comparison of the results presented in figure 6, it follows that in the time interval 1.0 s < τ < 1.05 s (fig. 6a), pressure oscillations occur both in the channel without local hydraulic resistance (1) and in the channel with local hydraulic resistance (2). But at τ > 2 s, pressure oscillations in the channel without local hydraulic resistance almost stop (fig. 6b, curve 1), while in the channel with local hydraulic resistance, pressure oscillations continue with a variable amplitude of ~3.0 Pa, ..., 3.5 Pa and with a frequency of ~240 Hz.

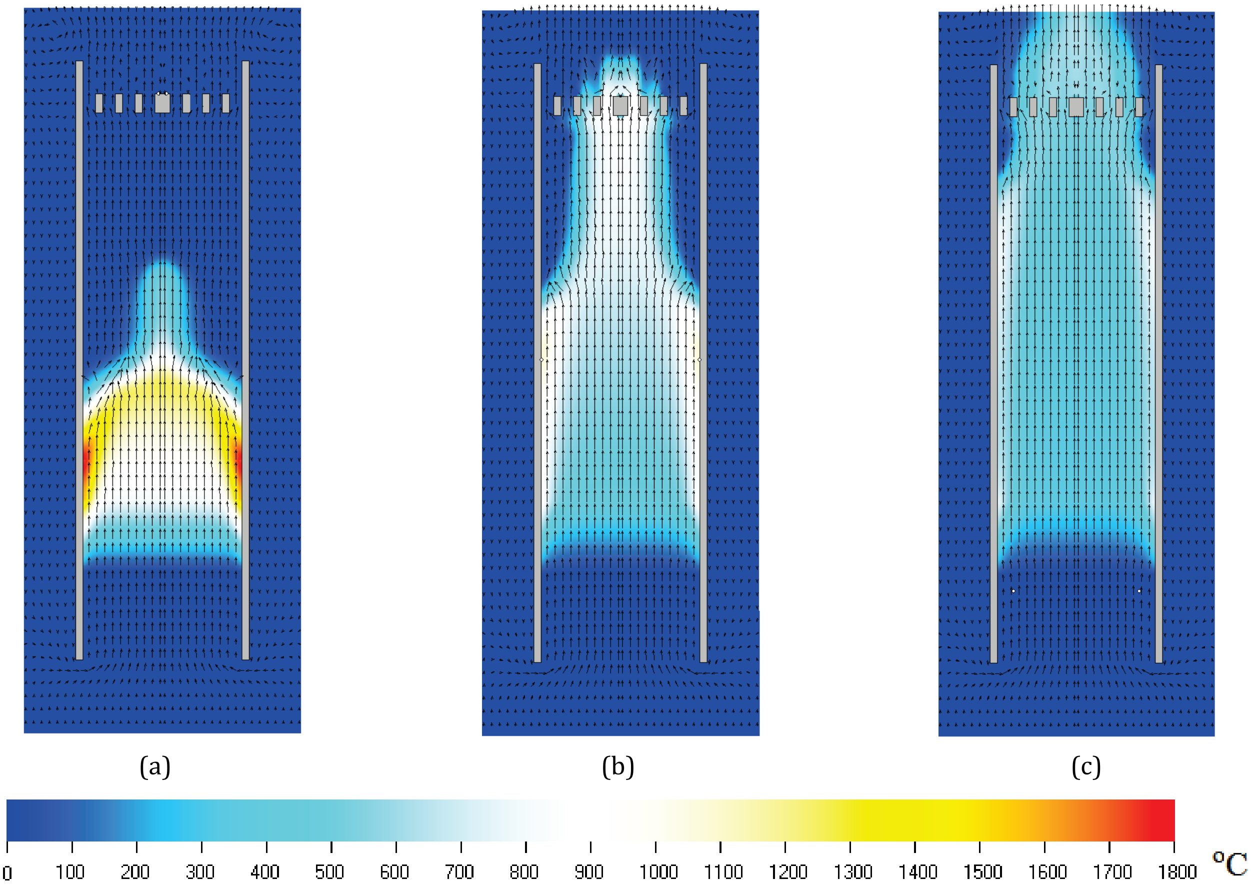


Figure 5. *Velocity and temperature fields in a vertical channel with local hydraulic resistance under conditions of heat supply with power Q = 3000 W: a) τ = 1.0 s; b) τ = 1.5 s; c) τ = 2.0 s*

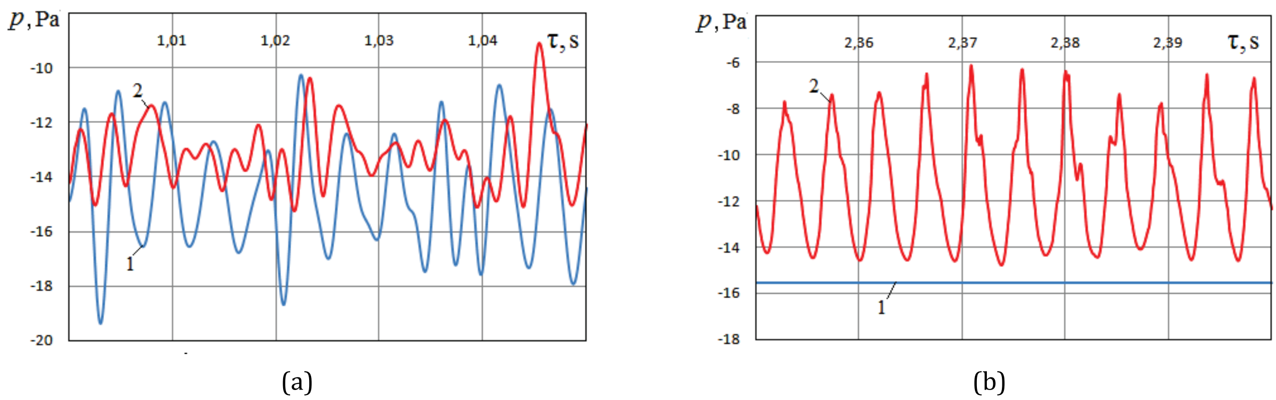
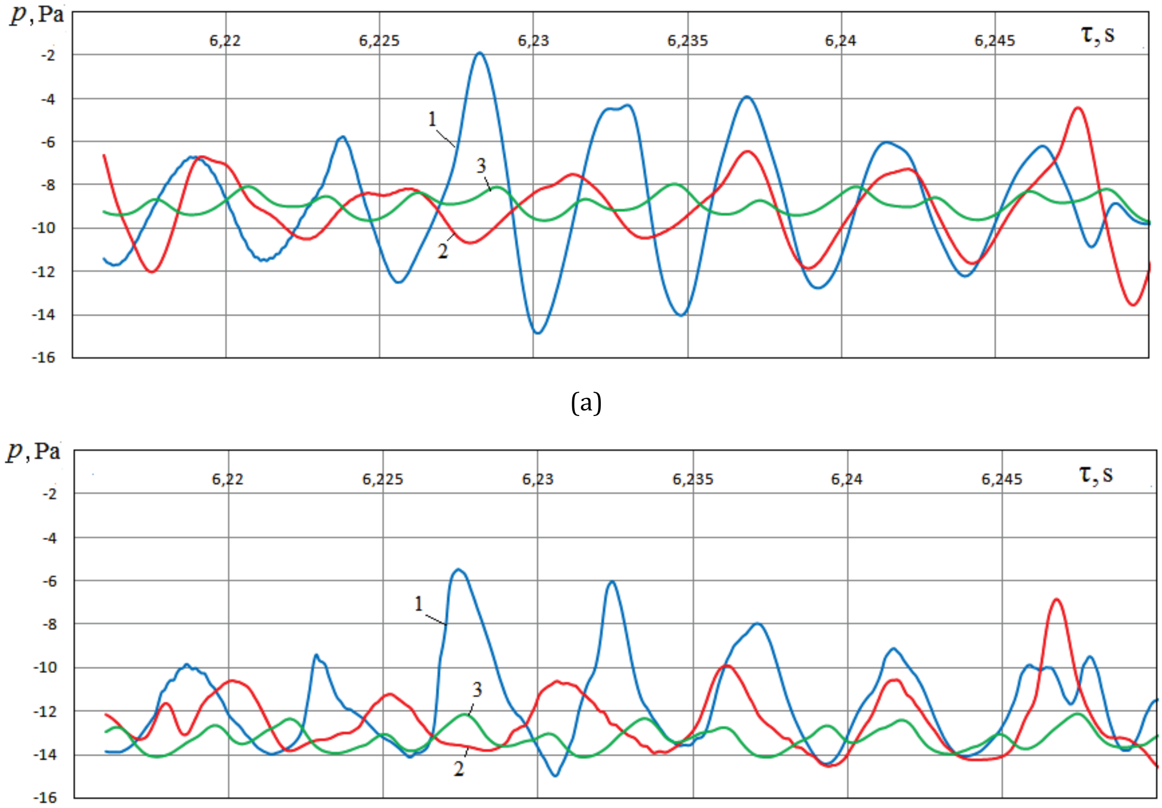


Figure 6. *Change in time of excess pressure in the cross section, located at a distance of 0.1 m from the outlet cross section of the channels with height H = 1.1 m without local hydraulic resistance (1) and with local hydraulic resistance (2) with local heat supply Q = 3000 W: a) 1.0 s < τ < 1.05 c; b) 2.35 s < τ < 2.4 s*

To determine the influence of the power of internal sources of heat release on the amplitude of pressure oscillations in a vertical channel with a height of H = 1.1 m with a local heat supply, the problem considered is solved also for Q = 2000 W and Q = 1000 W. The change in time of the excess pressure in the section, located at a distance of 0.07 m after the source of heat release (a) and in the section, which is located at a distance of 0.1 m from the outlet section of the channel (b) at three values of the power of the heat release source (Q = 3000 W; Q = 2000 W and Q = 1000 W) are shown in figure 7. The considered time interval is 6.215 s < τ < 6.25 s. As can be seen from this figure, with a decrease in the power of the heat release source, the amplitude of pressure oscillations also decreases in both cross sections of the channel. That is, an increase in the power of the heat source leads to an increase in the amplitude of pressure oscillations.

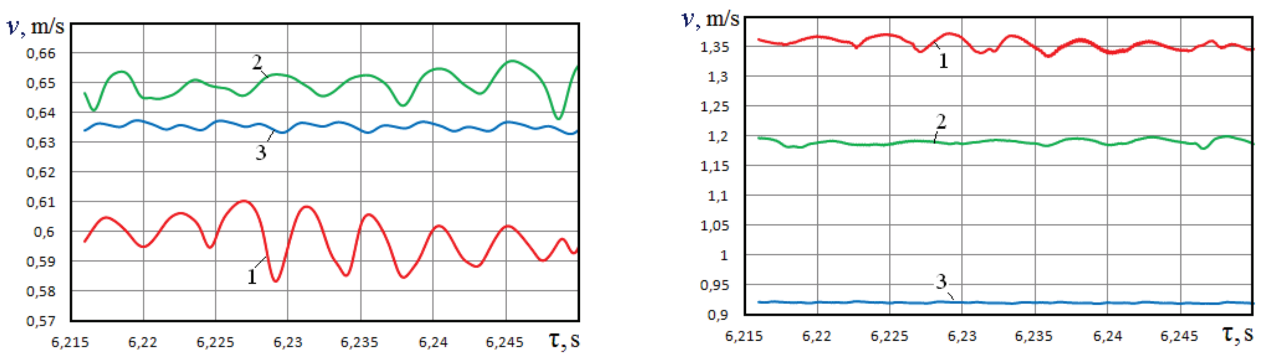


(a)

(b)

***FIGURE 7.** The change in time of the excess pressure in the cross section located at a distance of 0.07 m after the source of heat release (a) and in the section, located at a distance of 0.1 m (b) from the outlet section of the channel at different power of heat release: 1 – Q = 3000 W; 2 – 2000 W; 3 – 1000 W*

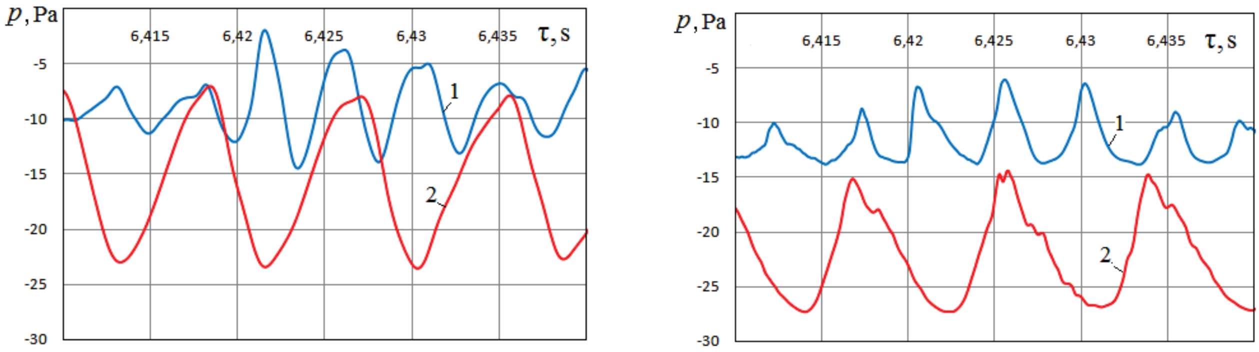
The influence of the power of internal sources of heat release on the time variation of the flow velocity in the inlet and outlet sections of the channel with height H = 1.1 m and diameter D = 0.1 m with local hydraulic resistance is shown in figure 8. As in figure 7, cases of three values of the power of sources of heat release are considered: Q = 3000 W; Q = 2000 W and Q = 1000 W. As can be seen from the figure, the average velocities in the inlet section of the channel are: 0.595 m/s at Q = 3000 W; 0.65 m/s at Q = 2000 W and 0.635 m/s at Q = 1000 W. In the outlet section of the channel, the velocities are: 1.35 m/s at Q = 3000 W; 1.19 m/s at Q = 2000 W and 0.92 m/s at Q = 1000 W. It follows from this that with an increase in the power of internal sources of heat release, the velocities in the outlet section of the channel increase. For the inlet section of the channel, such a pattern is not observed. Figure 8 also shows that the amplitudes of the velocity oscillations increase with the increase in the power of the heat generation sources.



(a) (b)

***FIGURE 8.** Change in velocity over time in the input (a) and output (b) cross sections of the channel at different capacities of local heat sources: 1 – Q = 3000 W; 2 – 2000 W; 3 – 1000 W*

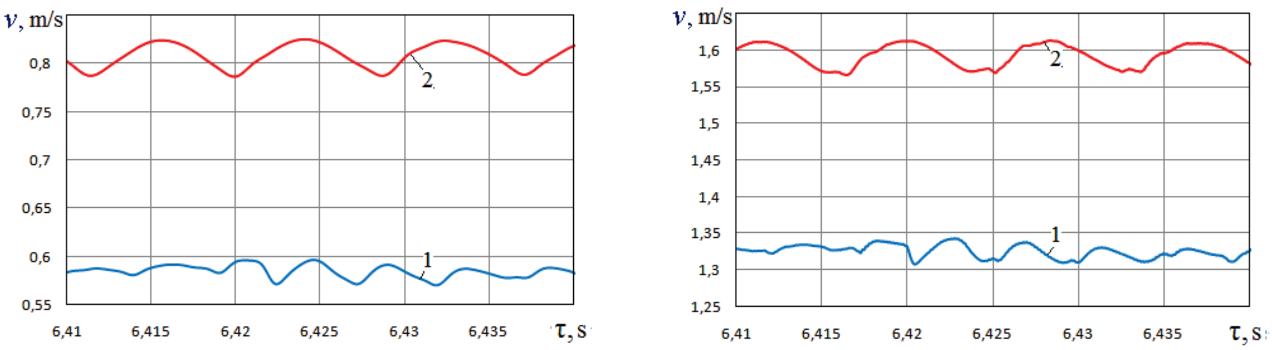
To determine the influence of the channel height on the characteristics of self-oscillations of pressure in the air flow, the problem was solved also for H = 2.0 m at Q = 3000 W. Other initial data for this case are the same as for H = 1.1 m. Based on the results of numerical modeling, the flow characteristics in channels of different heights are compared. Comparison of the nature of the change in time of excess pressure in the cross-sections of the channels that are located at a distance of 0.3 m from their inlet cross-sections, and in the cross-sections that are at a distance of 0.1 m from the outlet cross-sections of the channels are shown in figure 9. As can be seen from this figure, with the same power of heat generation sources, the amplitude and frequency of pressure oscillations at the same time intervals (6.41 s < τ < 6.44 s) will be different for channels of different heights. With an increase in the total height of the channel H and the same powers of the heat generation source, the amplitude of pressure oscillations in the above-mentioned sections of the channels increases. At the same time, the frequency of these oscillations decreases with increasing channel height. At H = 1.1 m, the frequency is ω_p ~ 240 Hz, and at H = 2.0 m, it is ω_p ~ 135 Hz.



(a) (b)

***FIGURE 9.** Change in time of excess pressure in the cross sections of the channels located at a distance of 0.3 m from the inlet sections (a) and in the sections located at a distance of 0.1 m (b) from the outlet sections at Q = 3000 W: 1– H = 1.1 m; 2 – H = 2.0 m*

The time variation of the flow velocities in the inlet and outlet sections of channels with a height of H = 1.1 m and H = 2.0 m at local heat sources power Q = 3000 W is shown in figure 10. As can be seen from the figure, the flow velocity, as well as the amplitude and frequency of velocity oscillations at the same time intervals (6.41 s < τ < 6.44 s) will be different for channels of different heights. With an increase in the height of the channel H, the average velocity also increases both in the inlet and in the outlet sections of the channel. This is due to the fact that as the height of the channel increases, the difference between the external pressure drop and the weight of the air in the channel grows. The amplitude of the velocity oscillations in the inlet and outlet sections of the channel increases with increasing channel height H, and the frequency of the velocity fluctuations decreases, as does the frequency of pressure oscillations.



(a) (b)

***FIGURE 10.** Change in velocity over time in the input (a) and output (b) cross-sections of the channel with local hydraulic resistance at the power of heat sources Q = 3000 W: 1– H = 1.1 m; 2 – H = 2.0 m*

Conclusions

With a local internal supply of heat to a compressible gaseous medium, which is located in a vertical channel, self-oscillations of pressure and velocity arise. Self-oscillations arising in a gas flow are one of the manifestations of flow instability. In the section of the channel, where the internal release of heat occurs, with the increase in temperature, the velocity of the free convection air flow in the channel also increases. The value of the velocity in the inlet section is always less than in the outlet section. Velocity oscillations in the inlet and outlet sections of the channel occur in antiphase, that is, at the minimum velocity in the inlet section of the channel, the velocity in the outlet section will be maximum.

Self-oscillations of pressure and velocity in a vertical channel with local heat release tend to decay with time. Self-oscillations do not damp and are maintained for a long time at a certain level in the presence of additional local hydraulic resistance in the channel. The nature of pressure self-oscillations is inharmonious with a non-constant amplitude.

The amplitude of pressure and velocity self-oscillations in a channel with local internal heat release increases with an increase in the power of heat release sources, as well as with an increase in the height of the channel. The oscillation frequency decreases with increasing channel height.

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