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A MATHEMATICAL MODEL BASED ON HEAT CONDUCTION EQUATION FOR PREDICTING AMORPHOUS MASSIVE STRUCTURES

Abstract: *The article presents the results of a study of the process of cooling a liquid molten metal on the surface of rotating rolls. A mathematical model is proposed to determine the optimal ratio of the cooling water temperature and the size of the cooling surfaces. The research results can be used to optimize technological processes associated with the production of thin metal sheets.*

Keywords*: cooling of melts, phase transformations, melt temperature.*

Introduction

Practical problem of determining the dynamics of solidification of the melt in time and prediction metal structure includes the calculation and analysis of temperature fields, determine the velocity of the solidification front in time, can be solved by means of mathematical modeling. For solving this problem mathematical model was developed in the basis, which was supposed to thermal conductivity equation. Formation a continuous layer of metal is complex irreversible process, consisting of a series of simple phenomena, which in this case cannot be considered without interacting with each other. Irreversibility of the process is associated primarily with the irreversibility of heat and mass transfer, the internal motion in the solid and liquid phases. In the general case a quantitative description of the process is based on the consideration of non-equilibrium thermodynamics uninsulated heterogeneous system consisting of few components, and the phases separated and the environment bounding hull.

Continuous process of pulling the metal layer can be represented as flows of viscous incompressible layer between two elastic-plastic surfaces (rollers), moving with a certain velocity (Fig. 1). Moreover, for an arbitrarily selected point in the cooling layer is characterized by a continuous change in temperature, pressure (stress), speed, distance to the boundary of the transition from liquid to solid state.

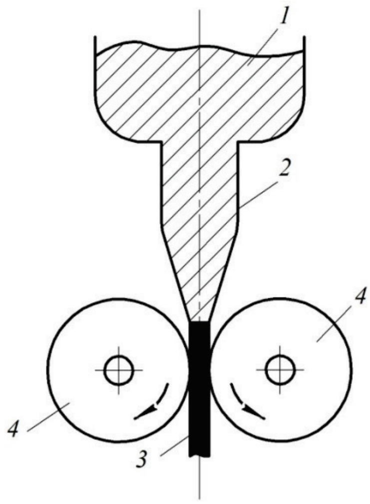

Figure 1. *Scheme of pulling process of a continuous layer of metal: 1 – the liquid metal melt; 2 – form for the metal; 3 – cooled metal layer; 4 – cooled rollers*

Thus, consideration of the cast layer (3) is bounded on both sides by curved surfaces (rollers) (4). Layer (3) is viscous incompressible fluid.

Formalization of the mathematical model

Since the problem of the motion of continuously cast layer is closely related to the problem of heat exchange, the key is to thermal conductivity equation in a general form for a moving continuous medium (layer 3) in which there are distributed sources of heat of the phase transition, depending on the specific heat of phase transition [1]:

$$\rho \cdot \frac{\partial}{\partial t}\left[c(T)T - L\Psi\right] = \mathrm{div}\left[\lambda(T)\mathrm{grad}\,T\right] + Q_v \tag{1}$$

and Fourier law thermal conductivity $q = -\lambda(T)\mathrm{grad}T$;

where:

ρ – density;

c – thermal capacity;

λ – thermal conductivity;

T – temperature;

L – specific heat of the phase transition (solidification);

Ψ – the proportion of solid phase (the liquid phase $\Psi = 0$, for the solid phase $\Psi = 1$, in the mushy zone $0 < \Psi < 1$);

$Q_v(x, y, z, t)$ – function characterizing the amount allocated during the solidification heat.

To obtain a unique solution of the problem is necessary to supplement its initial and boundary conditions [2, 3]:

$$\lambda_m \frac{\partial T}{\partial x} = 0 \tag{2}$$

$$x = \frac{H}{2} \;\; \lambda_m \frac{\partial T}{\partial x} = \alpha\left(T_m - T_a\right) \tag{3}$$

where:

T_m – metal temperature, °C;

T_a – air temperature, °C.

$$T(x, 0) = 1300\,^{\circ}\mathrm{C} \tag{4}$$

At the phase boundary solid metal – liquid melt is given by the boundary condition of Stephen:

$$\lambda_1 \frac{\partial T_1(t, \xi(t))}{\partial n} - \lambda_2 \frac{\partial T_2(t, \xi(t))}{\partial n} = L\frac{d\xi(t)}{dt} \tag{5}$$

where:

$\xi(t)$ – equation of the curve separating phases solid metal – liquid melt;

L – heat change of state, J/K (empirically determined value for the transition of the liquid melt in the solid metal);

n – normal to the curve;

$T_1(t,r)$ – temperature of the solid phase (solid metal);

$T_2(t,r)$ – temperature of liquid phase (liquid melt);

λ_1 – temperature diffusivity coefficient of the solid metal;

λ_2 – temperature diffusivity coefficient of the liquid melt.

Define the shape of the curve $\xi(t)$. We seek a solution of the thermal conductivity equation (1) in the following automodel form:

$$T(t,r)=f(z), \quad \text{where} \quad z=\frac{r}{\sqrt{t}} \tag{6}$$

Substituting (6) into (1) leads to the following ordinary differential equation:

$$-\frac{1}{2}f^{'}(z)\cdot z=\lambda\left(f^{''}(z)+\frac{1}{z}f^{'}(z)\right) \tag{7}$$

From which:

$$f(z)=C_1\int\frac{\exp\left(-\frac{z^2}{4\lambda}\right)}{z}dz+C_2 \tag{8}$$

where C_1 and C_2 are arbitrary constants of integration.

Next to finding the shape of the curve $\xi(t)$, we substitute (6) into the boundary condition Stephen (5). We get:

$$\lambda_1\frac{1}{\sqrt{t}}f_1^{'}\left(\frac{\xi(t)}{\sqrt{t}}\right)-\lambda_2\frac{1}{\sqrt{t}}f_2^{'}\left(\frac{\xi(t)}{\sqrt{t}}\right)=L\frac{d\xi}{dt}$$

from which:

$$\frac{\xi(t)}{\sqrt{t}}=\alpha=\text{const}$$

$$\lambda_1 f_1^{'}(\alpha)-\lambda_2 f_2^{'}(\alpha)=\frac{L}{2}\alpha \tag{9}$$

Consequently:

$$\xi(t)=\alpha\sqrt{t} \tag{10}$$

where the coefficient α is defined as the solution of the transcendental equation (9) with the known value of L aggregate heat transition molten liquid to the solid state.

Knowing the equation of the curve $\xi(t)$, separating the liquid melt phase – solid metal, we can reduce solution of original problem to the solution of the thermal conductivity equation with generalized (discontinuous) temperature conductivity coefficient:

$$\frac{\partial T}{\partial t}=\lambda(t,r)\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)$$

where:

$$\lambda(t,r)=\begin{cases}\lambda_1, \text{if } 0 \le r < \theta R+\xi(t) \\ \lambda_2, \text{if } \theta R+\xi(t) \le r < R\end{cases}$$

Initial conditions:

$$T\big|_{t=0}=T_m \quad at \quad 0 \le r < \theta R+\xi(t);$$
$$T\big|_{t=0}=T_a \quad at \quad \theta R+\xi(t) \le r < R$$

In Figures 2-4 shows graphs characterizing the dependence of the degree amorphization of the parameters of the pulling process (rapid casting) a metal layer between the cooling rolls.

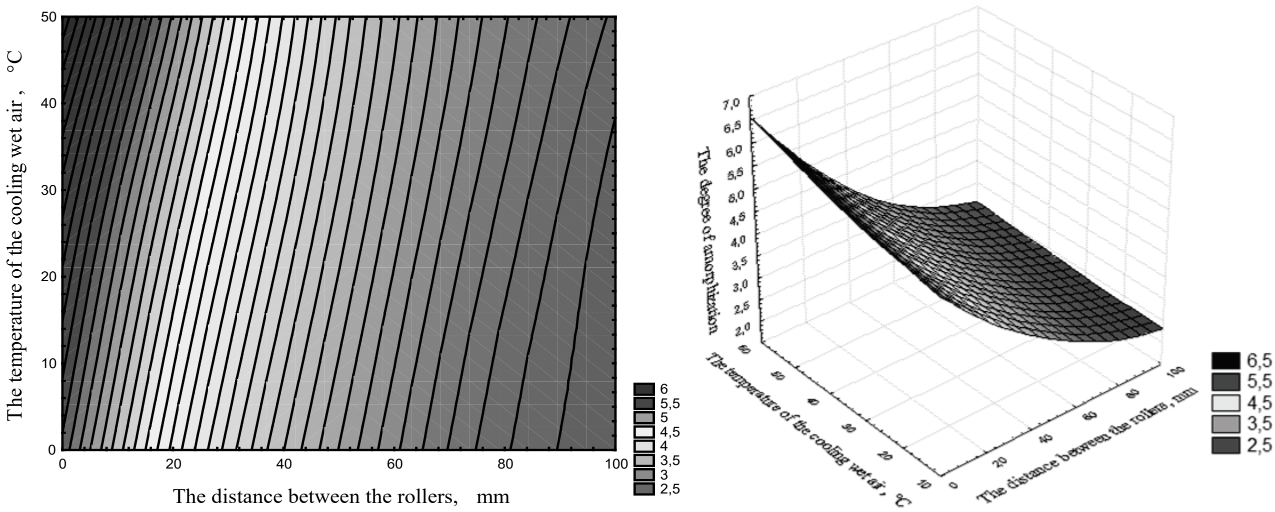

FIGURE 2. *Graph of the dependence of the degree of amorphization from the temperature of the cooling wet air and the distance between the rollers*

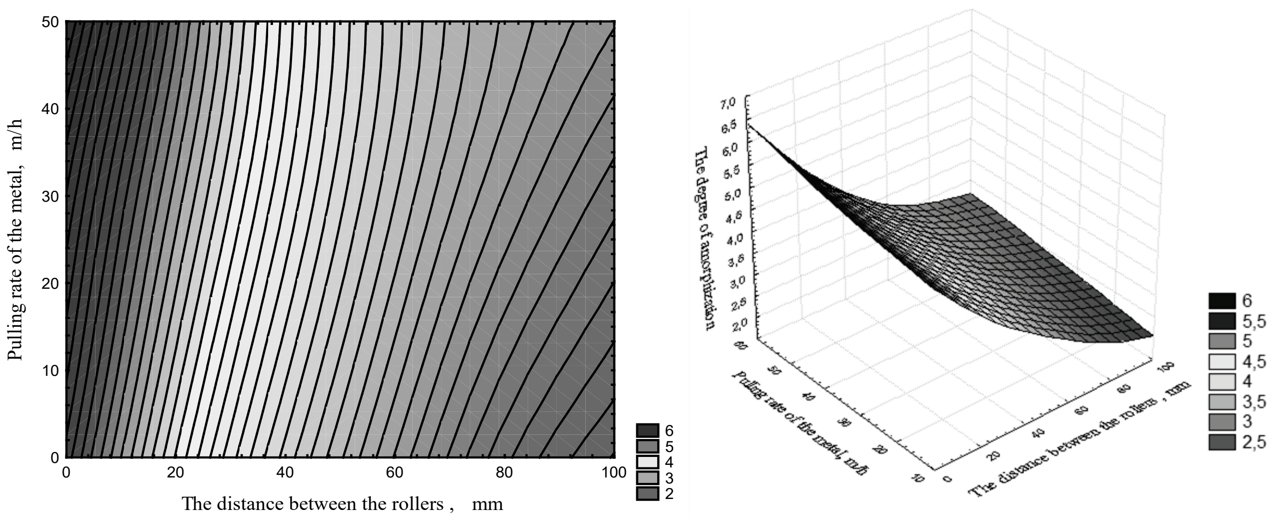

FIGURE 3. *Graph of the dependence of the degree of amorphization from pulling rate of the metal and the distance between the rollers*

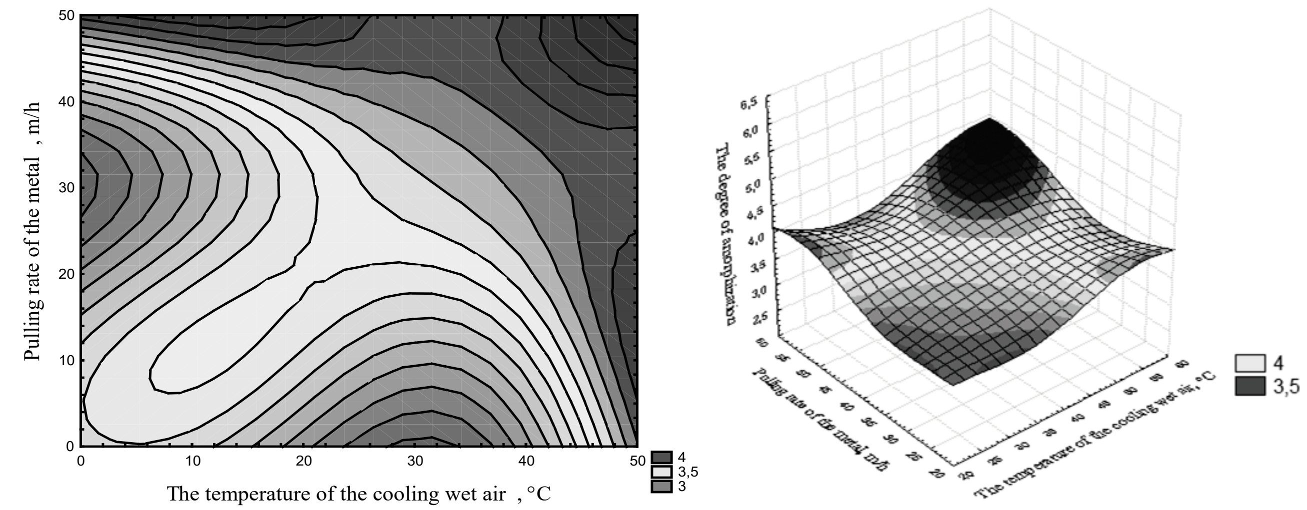

FIGURE 4. *Graph of the dependence of the degree of amorphization from pulling rate of the metal and the temperature of the cooling wet air*

Conclusions

1. Practically proved that under certain modes of the pulling process (rapid casting) metal layer between the cooling rolls is possible to obtain an amorphous structure at the layer boundaries.
2. To obtain an amorphous structure of the metal process pulling (fast casting) implement only when such interaction mode parameters (control factors) as the distance between the rollers, pulling speed of the metal and the temperature of the cooling wet air. To increase the degree of amorphization of the casting process is necessary to increase the temperature of the cooling wet air and pulling speed of the melt. But the main parameter of the process that has the greatest impact on the degree of amorphization of metal is the distance between the rollers. At the minimum values of distance degree of amorphization has a maximum value.

Conflicts of Interest: The author declares no conflict of interest.

References

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