3
2022
jntes

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Doi: 10.53412/jntes-2022-3-4

INFLUENCE OF STRUCTURAL CHARACTERISTICS OF POROUS MATERIALS ON THE COEFFICIENT OF THERMAL CONDUCTIVITY

Abstract: *The existing dependences of the effective coefficient of thermal conductivity of the material depending on the size and location of pores in it are analyzed and compared with each other and with previously obtained experimental data. It is shown that the resulting thermophysical properties of the material are affected not only by the porosity, but also by the location of the pores in the volume of the material. The disadvantages of the existing dependences of determining the effective thermal conductivity of the material on the type of porosity (both for porous material and for dispersed systems) are shown. Also, the most reliable dependences of the thermal conductivity coefficients on the porosity of dispersed systems for backfill materials and the need for their correction by empirical coefficients are determined.*
Complex indicators that fully describe the porous structure and on which the mathematical model of heat exchange processes in a porous medium should be based are proposed.

Keywords: *porous media, heat transfer, effective thermal conductivity coefficient, thermal resistance*

Introduction

Porous materials are widely used in industry. These are polystyrene foam materials, aerated concrete, some highly fire-resistant materials, foam glass, expanded clay. Porous materials have also occupied a niche in innovative technologies: combustion chambers, turbine walls, solar station receivers, thermal storage.

If we consider porous materials as dispersed systems, the main mechanism of heat transfer is, of course, the thermal conductivity of the material itself (the skeleton of the dispersed system) and the thermal conductivity of the medium in the pore (Hamdami et al., 2004). To calculate the thermal conductivity, a formal analogy between the basic laws of electricity and heat (the theory of generalized conductivity) is often used (Liang et al., 2013). However, it has been experimentally proved that when calculating the thermal conductivity of a porous material according to Maxwell's theory or Rayleigh's theory (calculation of the electric field of a system in which foreign particles of spherical shape are embedded), there is a significant discrepancy between the theoretically determined coefficient of thermal conductivity of a porous material and the actual one (Liang et al., 2013). Most likely, the discrepancy between theory and experiment is due to the idealization of model structures (Liang et al., 2013). There are also quite a number of other works (Säckel and Nieken, 2016; Lowell and Shields, 2016), in which it is stated that the generalized theoretical justification of the thermal conductivity of porous material gives differences with the experiments. Also, convection in the pores of thermal insulation products will depend on the operating conditions (Pavlenko, 2014). Therefore, it is necessary to understand how the porous structure affects the thermophysical properties of the material under different conditions of its operation, such as: temperature; dynamics of temperature change; duration of its exposure; humidity and others. Therefore, a working hypothesis of controlled structure formation of materials and the formation of its thermophysical properties was formed.

Working hypothesis: the influence of structural parameters on the thermophysical characteristics of the material allows to create a theoretical basis for the controlled structure formation of heat-insulating mats with specified thermophysical properties.

To confirm the working hypothesis, we consider the influence of structural parameters on the thermophysical characteristics of the material.

Statement of the problem

The thermal properties of thermal insulation materials and thermal protection structures will be affected by micropores, mesopores, macropores, cavities, voids and structural channels.

As mentioned above, as the pore size in materials and products increases above 0.1 mm, the effective thermal conductivity of the material due to convection increases. Heat transfer due to radiation with increasing optical path thickness decreases exponentially, but with increasing pore size, the radiation surface area also increases, which increases the radiation component (Zhang et al., 2015). Therefore, the effect of radiation on the flow of heat through the porous structure of thermal protection of power equipment when changing the shape and size of the pores is a dependence that should be considered as part of the generalized equation of the effective thermal conductivity of porous heat-insulating materials on the complex parameters of the porous structure. One of the complex indicators of the porous structure of the material or structure is the shape of the pore. The effective thermal conductivity coefficient for a material with pores oriented along the heat flow is almost twice as high as for a material with pores oriented perpendicular to the heat flow (Fugallo et al., 2014). This difference is the greater, the larger the pore size. Thus, the pore size is the second complex indicator of the porous structure.

Analysis of main studies and publications on the problem

Experimental data, both research and production, on optimum conditions of bloating are also different, since there is no theory generalizing the physical processes that occur during the formation of porosity. For example, preheating of bloating initial mixture of cellular glass to sintering temperature (690°C) is recommended as 70 minutes or 15 minutes.

All this proves that the porosity of the material plays an essential role in the thermophysical characteristics of the material.

As the pore size increases, the conduction of heat through convection increases. An increase in pore size has a similar effect on the radiative component.

Results of research

Let's take a closer look at the effect of pore location on the thermal conductivity of the material. Table 1 uses the following designations: porosity, λ_1 – heat transfer coefficient of the material (silica material with a heat transfer coefficient of 0.12 W/(m·K) was chosen as an example) λ_2 is the thermal conductivity coefficient of the medium (in the example selected air with gas admixtures having a thermal conductivity of 0.019 W/(m·K)). Heat flow is directed from the bottom upwards. The black color indicates the material.

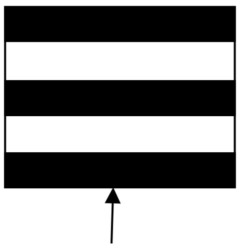
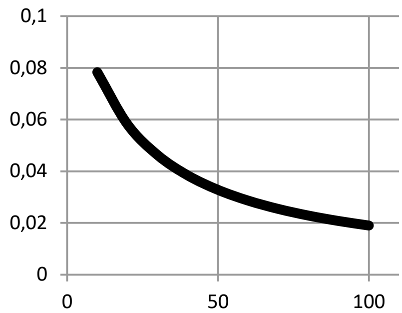
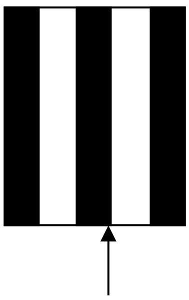
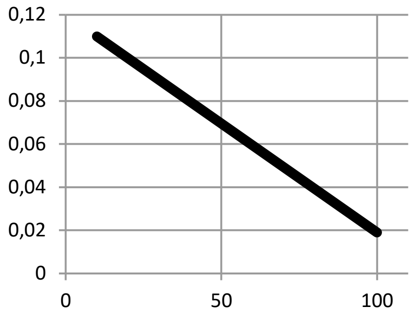
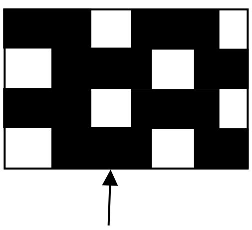
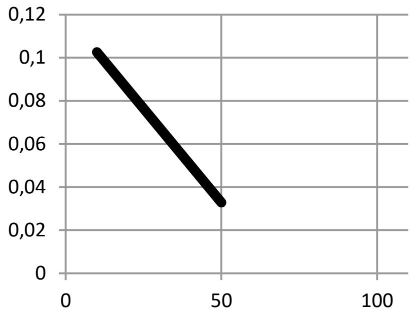
The formula derived by Aiken also applies to the calculation of the thermal conductivity of fill (No. 8 and No. 9 in Table 1):

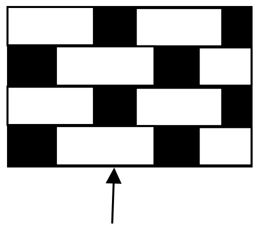
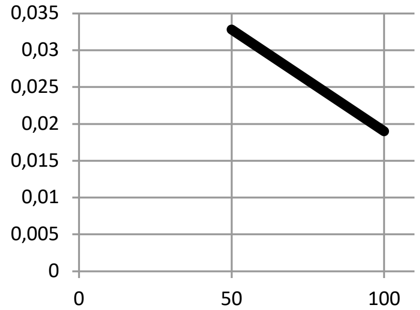
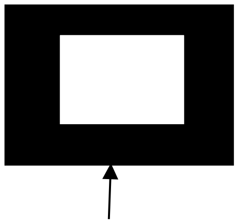
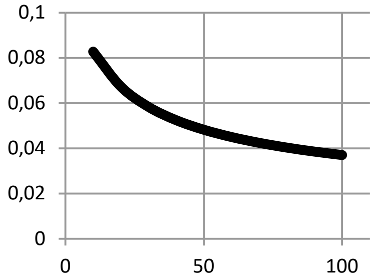
$$\lambda_{ef}=\frac{\lambda_1+1+\dfrac{2p\left(1-\dfrac{\lambda_1}{\lambda_2}\right)}{\dfrac{2\lambda_1}{\lambda_2}-1}}{1-p\dfrac{1-\dfrac{\lambda_1}{\lambda_2}}{\dfrac{2\lambda_1}{\lambda_2}+1}}$$

and Torcar's formula

$$\lambda_{ef}=\frac{\lambda_1}{1-p}$$

Table 1. *Summary of dependencies of the thermal conductivity coefficient on porosity for two-phase systems*

No.	Pore layout Formula for calculating the effective thermal conductivity coefficient	Pore layout Formula for calculating the effective thermal conductivity coefficient	Example $\lambda_{ef}=f(p)$
1		$\lambda_{ef}=\lambda_2\dfrac{100}{\dfrac{\lambda_2}{\lambda_1}(100-p)+p}$	
2		$\lambda_{ef}=\lambda_1\dfrac{100-p}{100}+\lambda_2\dfrac{p}{100}$	
3		if $p\leq 50\%$ $\lambda_{ef}=\lambda_2\left[\dfrac{4p}{1+\dfrac{\lambda_2}{\lambda_1}}+\dfrac{\lambda_1}{\lambda_2}(1-2p)\right]$	

<table>
<tr><td>4</td><td></td><td>if $p \ge 50\%$
$$\lambda_{ef} = \lambda_2 \left[\frac{4(1-p)}{1+\frac{\lambda_2}{\lambda_1}} + (2p-1) \right]$$</td><td>
</td></tr>
<tr><td>5</td><td></td><td>$$\lambda_{ef} = \frac{\lambda_1^2 p^{\frac{2}{3}} + \lambda_1(\lambda_2-\lambda_1)}{\lambda_1 + p^{\frac{2}{3}}(\lambda_2-\lambda_1)}$$</td><td>
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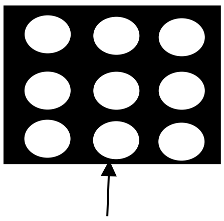
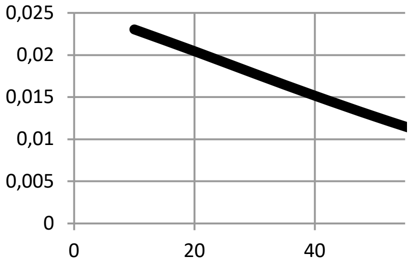
<tr><td>6</td><td></td><td>if $p \le 50\%$
$$\lambda_{ef} = \frac{\lambda_2 p + \frac{\lambda_2}{\lambda_1}\left(1-p^{\frac{2}{3}}\right)}{p - p^{\frac{2}{3}} \frac{\lambda_1}{\lambda_2}\left(1-p^{\frac{2}{3}}+p\right)}$$</td><td>
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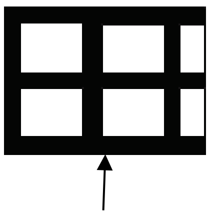
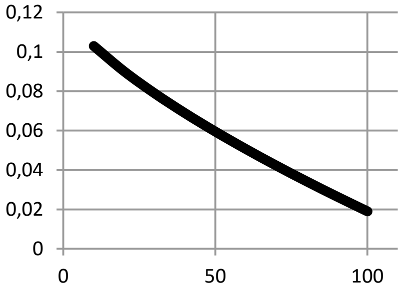
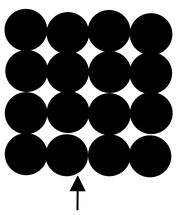
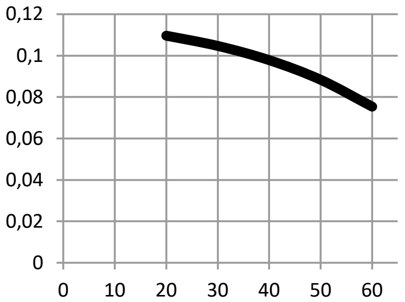
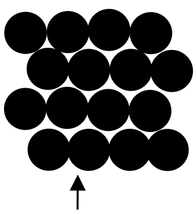
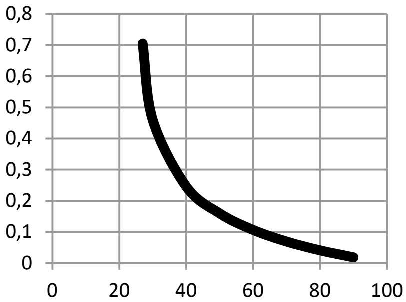
<tr><td>7</td><td></td><td>$$\lambda_{ef} = \lambda_2 p^{\frac{1}{3}} + \lambda_1 (1-p)^{\frac{2}{3}}$$</td><td>
</td></tr>
<tr><td>8</td><td></td><td>$p \approx 48\%$
$$\lambda_{ef} = \frac{1.5\pi\lambda_1(0.9-p)}{(2.1-p)^2}$$</td><td>
</td></tr>
<tr><td>9</td><td></td><td>$p \approx 30\%$
$$\lambda_{ef} = 3\pi\lambda_1 \ln \frac{43+0.31p}{p-26}$$</td><td>
</td></tr>
</table>

Aiken's formula gives the smallest error for particle shapes approaching a sphere and for porosity less than 50%. For dispersed material backfill, case No. 9 in Table 1 is the most appropriate.

For granular silica backfills it is recommended to use the Odelevsky formula:

$$\lambda_{ef} = \lambda_1 \left(1 + \frac{p_1}{\frac{1-p_2}{3} + \frac{\lambda_1}{\lambda_2 - \lambda_1}} \right)$$

Using Table 1, graphical analysis of formulas for calculating the thermal conductivity of porous materials (Fig. 1) and separately for the backfill (Fig. 2). For the porous material we leave the same values of thermal conductivity as in the example, and for the backfill of solid phase we take the thermal conductivity equal to granules of thermal insulation material based on silica for medium-temperature insulation (0.036 W/(m·K)) (Pavlenko, 2020).

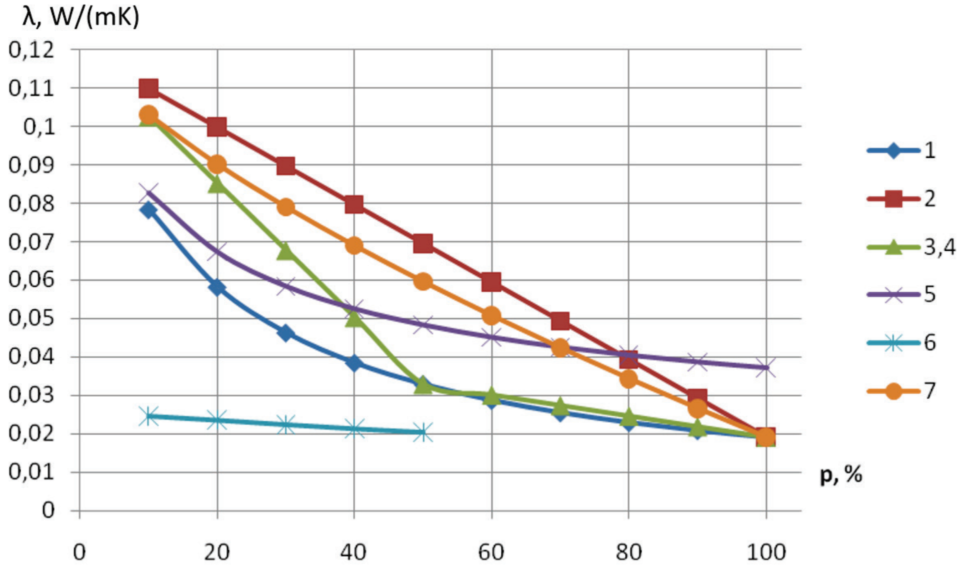


FIGURE 1. *Influence of porosity on the effective heat transfer coefficient for different schemes of porosity according to Table 1*

Structures 1 and 2 are given as the maximum and minimum teleretic values of the effective thermal conductivity and set the limits of the study.

As can be seen from Figure 1, the structure No. 6 with the calculated formula underestimates the value of the thermal conductivity coefficient, and at a porosity of 10% the thermal conductivity coefficient of the material cannot be approximately equal to the thermal conductivity coefficient of air. Therefore, the calculation formula No. 6 is not correct for these ratios of the coefficients of thermal conductivity of air and material.

Calculation formula No. 5 does not take into account the influence of pore size, or rather it was considered by the author as a large structure. Also, contrary to logic, the values of thermal conductivity in No. 5 are underestimated at porosity up to 30%.

As can be seen from Figure 2, the formula of Aiken and Bogomolov (No. 9 of Table 1) shows only a general dependence and needs empirical correction. The Torkar and Odelevsky dependencies express an increase in the coefficient of thermal conductivity with an increase in porosity and are suitable only for moist disperse systems or systems with large pore sizes. Nekrasov's dependence for an idealized structure (No. 8 of Table 1) is the most acceptable.

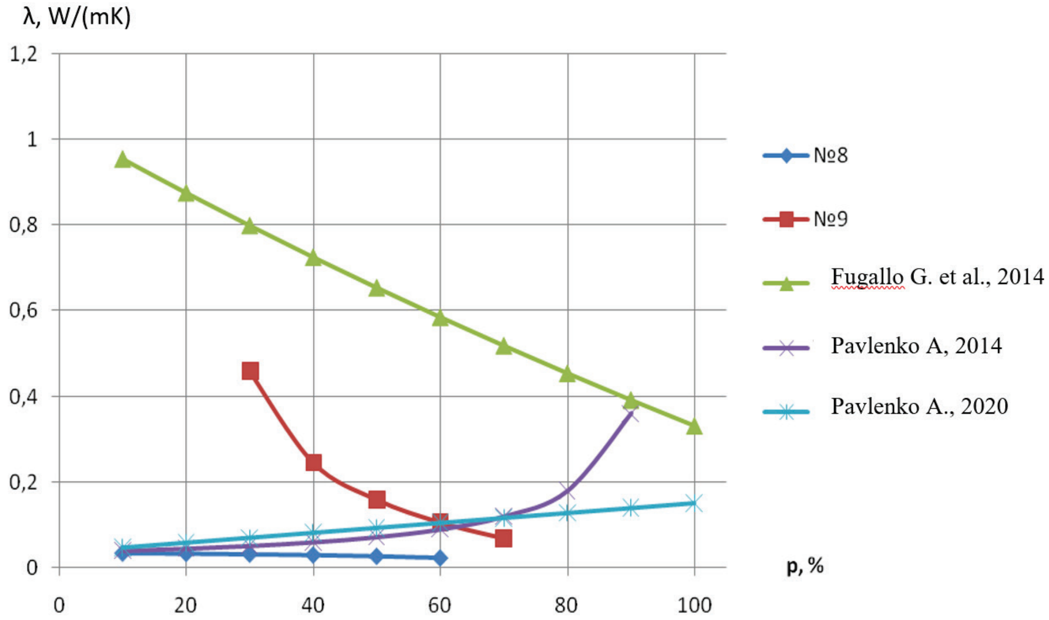


FIGURE 2. *Effect of porosity on the effective coefficient of thermal conductivity for fillings of dispersed material according to Table 1*

The formula for calculation of channel porosity in absorbers was also derived. The mathematical model of the proposed equation was based on a conditional isothermal channel with additional resistance to fluctuations in the direction of the dislocation vector.

Let us calculate the resistance of displacement vectors by the method of electrothermal analogy, for two-dimensional space in the direction of the structure vectors for a chaotic structure of the first order. In this case, we assume that the *x* direction in which the heat flow passes in the higher scale structure can be decomposed into *x* and *y* vectors in the lower scale structure:

$$Rxj = \sum_{i=1}^{n_1} Rxj_i \,, j = 1,2$$

$$Rxj_i = Rf + Rs$$

$$R_{x1} = n_x \frac{d_x}{\lambda_f} + \frac{n_x \left(r_x - d_x\right)}{\lambda_s}$$

$$R_{x2} = n_y \frac{d_2}{\lambda_f} + \frac{n_y \left(r_y - d_2\right)}{\lambda_s}$$

$$R_{x1} = \frac{n_x d_x \lambda_s + \lambda_f k_x l_x - \lambda_f n_x d_x}{\lambda_s \lambda_f}$$

$$R_{x2} = \frac{n_y d_2 \lambda_s + \lambda_f k_y l_y - \lambda_f n_y d_2}{\lambda_s \lambda_f}$$

$$\frac{1}{R} = \frac{1}{R_{x1}} + \frac{1}{R_{x2}}$$

So

$$R=\frac{\left(\frac{n_x d_x\lambda_s+\lambda_f k_x l_x-\lambda_f n_x d_x}{\lambda_s\lambda_f}\right)\left(\frac{n_y d_2\lambda_s+\lambda_f k_y l_y-\lambda_f n_y d_2}{\lambda_s\lambda_f}\right)}{\left(\frac{n_x d_x\lambda_s+\lambda_f k_x l_x-\lambda_f n_x d_x}{\lambda_s\lambda_f}\right)+\left(\frac{n_y d_2\lambda_s+\lambda_f k_y l_y-\lambda_f n_y d_2}{\lambda_s\lambda_f}\right)}$$

$$=\frac{\left(n_x d_x\lambda_s+\lambda_f k_x l_x-\lambda_f n_x d_x\right)\left(n_y d_2\lambda_s+\lambda_f k_y l_y-\lambda_f n_y d_2\right)\left(\frac{1}{\lambda_s\lambda_f}\right)}{n_x d_x\lambda_s+\lambda_f k_x l_x-\lambda_f n_x d_x+n_y d_2\lambda_s+\lambda_f k_y l_y-\lambda_f n_y d_2}$$

$$=\frac{\left(n_x d_x\lambda_s+\lambda_f k_x l_x-\lambda_f n_x d_x\right)\left(n_y d_2\lambda_s+\lambda_f k_y l_y-\lambda_f n_y d_2\right)}{\lambda_s\lambda_f\left(n_x d_x\lambda_s+\lambda_f k_x l_x-\lambda_f n_x d_x+n_y d_2\lambda_s+\lambda_f k_y l_y-\lambda_f n_y d_2\right)}$$

If there is no displacement in the *x*-axis $k_x=1$

$$R=\frac{\left(n_x d_x\lambda_s+\lambda_f l_x-\lambda_f n_x d_x\right)\left(n_y d_2\lambda_s+\lambda_f k_y l_y-\lambda_f n_y d_2\right)}{\lambda_s\lambda_f\left(n_x d_x\lambda_s+\lambda_f l_x-\lambda_f n_x d_x+n_y d_2\lambda_s+\lambda_f k_y l_y-\lambda_f n_y d_2\right)}$$

For an open porous structure will be true $n_x d_x=l_x$

$$R=\frac{\left(l_x\lambda_s+\lambda_f l_x-\lambda_f l_x\right)\left(n_y d_2\lambda_s+\lambda_f l_y k_y-\lambda_f n_y d_2\right)}{\lambda_s\lambda_f\left(n_x d_x\lambda_s+\lambda_f l_x-\lambda_f l_x+n_y d_2\lambda_s+\lambda_f k_y l_y-\lambda_f n_y d_2\right)}$$

Porosity in the direction of the radius vector

$$\varnothing'=\frac{n_y d_2}{k_y l_y}$$

$$R=\frac{l_x\left(\lambda_s\varnothing' k_y l_y+k_y l_y\lambda_f\left(1-\varnothing'\right)\right)}{\lambda_f\left(\lambda_s\left(n_x d_x+\varnothing' k_y l_y\right)+\lambda_f k_y l_y\left(1-\varnothing'\right)\right)}$$

$$R=\frac{l_x\left(\lambda_s\varnothing'+\lambda_f\left(1-\varnothing'\right)\right)}{\lambda_f\left(\lambda_s\left(\frac{n_x d_x}{k_y l_y}+\varnothing'\right)+\lambda_f\left(1-\varnothing'\right)\right)}$$

$$\lambda_{vector\_f}=\lambda_f\frac{\lambda_s\frac{l_x}{k_y l_y}+\lambda_s\varnothing'+\lambda_f\left(1-\varnothing'\right)}{\lambda_s\varnothing'+\lambda_f\left(1-\varnothing'\right)}$$

$$\lambda_{vector\_f}=\lambda_f\left[1+\frac{\lambda_s\frac{l_x}{k_y l_y}}{\lambda_s\varnothing'+\lambda_f\left(1-\varnothing'\right)}\right]$$

The above formula requires similar transformations with the thermal conductivity of a solid. After that, it is necessary to convert the *xy* space into a single vector of the heat flux direction to obtain the desired function.

Complex indicators of porous structure

Most of the existing studies of porous structures of materials for thermal protection of elements of industrial power plants take into account the total porosity as the main structural characteristic of the thermal insulation material, and sometimes take into account either the shape of the pores and their number or the type of pores (Rudobashta et al., 2015). The analysis of the current literature shows that even the simultaneous consideration of the total porosity of the material, the size and type of pores is not enough to fully characterize the porous structure of the heat-insulating material. In (Rudobashta et al., 2015), it is proposed to pay attention to the following main factors in porous systems: the nature of the structure, the number of structure components, the aggregate state of the structure components and the processes of interaction between the structure components. These complex indicators are convenient for the separation of porous systems as a whole because they allow controlling the thermophysical properties of a particular macroporous material by changing the porosity structure. But these indicators do not allow to find the functional dependence of the thermophysical properties of heat-insulating materials on the porous structure, which does not allow to optimize the thermophysical properties of porous heat-insulating material by creating predictable porous structures. Therefore, we propose the main complex indicators of the porous structure of the heat-insulating material and structures of thermal protection of elements of industrial power plants, which fully reflect the porous structure and make it possible to draw up a regression equation for the dependence of the thermal properties of porous heat-insulating materials on the proposed indicators:

1. Porosity *P*, % – porosity as a general indicator of the density of thermal insulation material and thermal protection structures.
2. Number of pores *n*, pcs/m^3 – the number of pores for a homogeneous structure in combination with porosity gives a general idea of the distribution of pores in the material. The change in the number of pores over time during the formation of the porous structure of heat-insulating materials expresses the dynamics of the pore formation process.
3. The location of the pores in space – described by the Brave translation system (Brave lattice), in which the pore is the core of the lattice with dimensions smaller than the Wigner-Seitz cell, or the statistical distribution of pores in the volume of the insulating material.
4. Pore shape – a spatial coordinate function describing the shape of the pore. It is possible to accept the description of all pores as spheres with the description of the deformation inherent in this sphere, according to the Poincaré hypothesis, or the overall dimensions of the pore, or the general coefficient of geometric characteristics of the porous structure.
5. Indicators of the gas state in the pores – the temperature gradient on which convection in the pores and the physical properties of the coolant in the pore depend. It can also be represented by the product of Grasgoff number and Prandtl number.
6. Specific surface area porosity *S*.

To determine the energy intensity of the created porous thermal insulation materials and structures of thermal protection of elements of industrial power plants, the energy of formation of the porous structure is used:

$$dQ_{por} = T_{por} dS + \varphi_{por} dM_{por}$$

Conclusion

The closest to minimally thermally conductive (theoretical) structure is the staggered arrangement of pores stretched perpendicular to the heat flow over the volume. This is also confirmed by experimental studies.

After analyzing Figure 2 it can be concluded that for the idealized case of fill with total porosity of about 50% the effective thermal conductivity coefficient can be calculated with the help of Nekrasov's dependence (No. 8 of Table 1). For practical calculations (porosity of disperse system is about 30%), it is necessary to correct Bogomolov's dependence by empirical coefficients (No. 9 of Table 1).

Complex indicators that fully describe the porous structure and on which the mathematical model of heat exchange processes in a porous medium should be based are proposed: porosity, number of pores, the location of the pores in space, pore shape, indicators of the gas state in the pores.

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